

Problem 2.37

[Difficulty: 2]

2.37 The variation with temperature of the viscosity of air is represented well by the empirical Sutherland correlation

$$\mu = \frac{bT^{1/2}}{1 + S/T}$$

Best-fit values of b and S are given in Appendix A. Develop an equation in SI units for kinematic viscosity versus temperature for air at atmospheric pressure. Assume ideal gas behavior. Check by using the equation to compute the kinematic viscosity of air at 0°C and at 100°C and comparing to the data in Appendix 10 (Table A.10); plot the kinematic viscosity for a temperature range of 0°C to 100°C, using the equation and the data in Table A.10.

Given: Sutherland equation

Find: Corresponding equation for kinematic viscosity

Solution:

Governing equation: $\mu = \frac{b \cdot T^{\frac{1}{2}}}{1 + \frac{S}{T}}$ Sutherland equation $p = \rho \cdot R \cdot T$ Ideal gas equation

Assumptions: Sutherland equation is valid; air is an ideal gas

The given data is $b = 1.458 \times 10^{-6} \cdot \frac{\text{kg}}{\text{m} \cdot \text{s} \cdot \text{K}^{\frac{1}{2}}}$ $S = 110.4 \cdot \text{K}$ $R = 286.9 \cdot \frac{\text{J}}{\text{kg} \cdot \text{K}}$ $p = 101.3 \cdot \text{kPa}$

The kinematic viscosity is $\nu = \frac{\mu}{\rho} = \frac{\mu \cdot R \cdot T}{p} = \frac{R \cdot T}{p} \cdot \frac{b \cdot T^{\frac{1}{2}}}{1 + \frac{S}{T}} = \frac{R \cdot b}{p} \cdot \frac{T^{\frac{3}{2}}}{1 + \frac{S}{T}} = \frac{b' \cdot T^{\frac{3}{2}}}{1 + \frac{S}{T}}$

where $b' = \frac{R \cdot b}{p}$ $b' = 4.129 \times 10^{-9} \cdot \frac{\text{m}^2}{\text{K}^{1.5} \cdot \text{s}}$

$$b' = 286.9 \cdot \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \times 1.458 \times 10^{-6} \cdot \frac{\text{kg}}{\text{m} \cdot \text{s} \cdot \text{K}^{\frac{1}{2}}} \times \frac{\text{m}^2}{101.3 \times 10^3 \cdot \text{N}} = 4.129 \times 10^{-9} \cdot \frac{\text{m}^2}{\text{s} \cdot \text{K}^{\frac{3}{2}}}$$

Hence $\nu = \frac{b' \cdot T^{\frac{3}{2}}}{1 + \frac{S}{T}}$ with $b' = 4.129 \times 10^{-9} \cdot \frac{\text{m}^2}{\text{s} \cdot \text{K}^{\frac{3}{2}}}$ $S = 110.4 \text{ K}$

Check with Appendix A, Table A.10. At $T = 0^\circ\text{C}$ we find $T = 273.1\text{ K}$ $\nu = 1.33 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$

$$\nu = \frac{4.129 \times 10^{-9} \frac{\text{m}^2}{\text{s} \cdot \text{K}^{\frac{3}{2}}} \times (273.1 \cdot \text{K})^{\frac{3}{2}}}{1 + \frac{110.4}{273.1}} \quad \nu = 1.33 \times 10^{-5} \frac{\text{m}^2}{\text{s}} \quad \text{Check!}$$

At $T = 100^\circ\text{C}$ we find $T = 373.1\text{ K}$ $\nu = 2.29 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$

$$\nu = \frac{4.129 \times 10^{-9} \frac{\text{m}^2}{\text{s} \cdot \text{K}^{\frac{3}{2}}} \times (373.1 \cdot \text{K})^{\frac{3}{2}}}{1 + \frac{110.4}{373.1}} \quad \nu = 2.30 \times 10^{-5} \frac{\text{m}^2}{\text{s}} \quad \text{Check!}$$

